Models and Precision:

The Quality of Ptolemy's Observations and Parameters

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APPENDIX 1

Secular Accelerations of the Sun and Moon

The foregoing investigation draws importantly on the evidence of the errors in Ptolemy's solar and lunar observations that is obtained from comparisons with modern theory. Consequently, it is desirable to minimize the possibility of introducing significant systematic errors from modern theory into the results. The inequalities in the motions of the Sun and Moon are presently known with far greater accuracy than such comparisons require. The modern values for the mean longitudes of the Sun and Moon at ancient epochs, however, are affected by considerable uncertainty as to the magnitudes of the secular accelerations of the mean motions of both celestial bodies.

This uncertainty arises primarily from the apparent difference between the results obtained from analyses of modern observations and those derived from ancient observations. It is also, however, reflected in the different results derived from investigations of ancient observations—differences which arise partly from divergent evaluations of the quality of the empirical evidence from antiquity and partly from variations in the observations investigated, methodologies employed, assumptions made, and even errors committed. Finally, a small but additional element of uncertainty arises from the use by the various investigators of slightly different elements—and, hence, of different effective epochs—thus complicating the comparison of their results.[1]

The following discussion reviews the principal attempts to determine the accelerations of the Sun and Moon down to the 'definitive' determination by Spencer Jones [1939], which has been adopted in national ephemerides (i.e., 'modern theory') since 1952. Its purpose is to identify the values of these parameters least likely to introduce significant errors into comparisons of Ptolemy's observations with modern theory.

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[1] Fotheringham's researches [1909, 1915a, 1918, 1920, and 1923] are a particularly troublesome example of these difficulties, since nearly all are based on different lunar elements.

My principal finding is that de Sitter's [1927] ostensibly definitive analysis of the ancient observational evidence, which Jones [1939] incorporated in his determination, was seriously flawed by several significant errors, the correction of which causes the apparent difference between the accelerations derived from ancient and modern observations to disappear. This correction leads to a significantly smaller value of the Moon's apparent non-gravitational acceleration ($+3.6''$) than that ($+5.22''$) currently used by the Nautical Almanac Offices [1961, 98, 107], and to a slightly smaller value ($+1.1''$) of the Sun's apparent acceleration than is presently accepted ($+1.23''$). These values are also smaller than those found by Schoch and adopted by P. V. Neugebauer [1929, 1934] and Tuckerman [1962–1964] in their tables. They are also significantly different from those derived from ancient observations by Newton [1969, 1970] and Muller and Stephenson [1975], but are consistent with the results obtained from ancient observations by Curott [1966] and from modern observations by Morrison and Ward [1975]. Moreover, a recent analysis of ancient and medieval observations by Stephenson and Morrison [1984], which includes extensive data from cuneiform sources, suggests accelerations for the period covered by Ptolemy's observations which are only slightly higher than those used here, although lower than those of Fotheringham, Schoch, and, most recently, Newton [1985].

To facilitate the comparison of historical investigations, I have followed the convention of using the term 'acceleration' to denote the coefficient of the term in T^2 in the polynomial expression for any element, where T is expressed in Universal (rather than Ephemeris) Time. Thus, except where otherwise noted, the accelerations referred to denote the apparent accelerations resulting from both gravitational and non-gravitational causes. The symbols used in equations are as follows:

- S_m Sidereal lunar acceleration in longitude
- S'_m Non-gravitational lunar acceleration in longitude, $S_m - 6.05''$
- S_s Sidereal, non-gravitational apparent solar acceleration in longitude due to the slowing of the Earth's rate of rotation
- S_D Acceleration of the Moon's mean elongation, $S_m - S_s$
- S'_D Non-gravitational acceleration in elongation, $S'_m - S_s = S'_D - 6.05''$.

Ever since Clemence's paper [1948, 172], it has been customary to use Ephemeris Time as the independent variable and to consider $\Delta T = ET - UT$ (the cumulative effect of the Earth's variable rotation) in place of S_s, and to use $1/2\,\dot{\eta}_m$ (the resulting non-gravitational retardation of the Moon's sidereal longitude) in place of S'_m. To facilitate comparisons with recent studies, I note the following relationships between the accelerations discussed here and related parameters discussed by others. (The approximate relationship for S'_m results from the adoption of different effective epochs for the modern mean motions.)

$$S_s = \frac{\Delta T}{24.35T^2} = \frac{1/2\dot{e}}{24.35} = \frac{\dot{w}/w_e}{15.46}$$
$$S'_m = \frac{\mu_m}{\mu_s}S_s + \frac{1}{2}\dot{\eta}_m \cong 13.168 S_s + \frac{1}{2}\dot{\eta}_m.$$

Early determinations of the Moon's acceleration

The first to suggest that the Moon exhibited a sensible acceleration was Edmund Halley. On October 19, 1692, he read a paper before the Royal Society proposing that certain discrepancies among the terrestrial longitudes ascribed to such places as Babylon and Antioch could be reconciled by supposing that the Moon (and planets) were retarded by the aether [MacPike 1932, 229].[2] This retardation, Halley concluded, showed the impossibility of the world's eternity. Subsequently, on October 18, 1693, he promised [MacPike 1932, 232]

> to make out the necessity of the world's coming to an end, and consequently that it must have had a beginning, which hitherto had not been evinced from anything that has been observed in nature.

Although the *Journal Book* of the Royal Society [see MacPike 1932, 232] notes that Halley was ordered to print a dissertation on this subject, his only published reference to the Moon's acceleration appeared in 1695 as a postscript to an article discussing the ruins of Palmyra [Halley 1695, 174].[3]

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[2] MacPike [1932, 210] has collected the references to Halley in Thomas Birch's *History of the Royal Society*, which includes the contents of the Society's *Journal Book* up to December 1687. MacPike also published further references to Halley from the Society's *Journal Book* from January 1687/8 to July 1, 1696. The quotations in the text are from this source.

[3] One frequently encounters the statement that Halley first proposed the existence of a lunar acceleration in an earlier paper published in 1693 [Halley 1693], in which he discusses four eclipses described by al-Battānī and corrects some of the numbers given in the two editions of al-Battānī [1537, 1645] then available. Although it is possible that his discovery of the acceleration arose from comparing his reconstructed epochs for al-Battānī's lunar arguments with values computed from contemporary lunar theory, Halley makes no mention of the phenomenon in this paper. Cf. Houzeau and Lancaster 1882–1889, ii col. 1197.

In this he asked 'any curious traveller residing there' to make observations of lunar eclipses in Baghdad, Aleppo, and Alexandria, so as to enable him to re-determine the longitudes of these places. With secure values for these longitudes, he

> could then pronounce in what proportion the Moon's motion does accelerate; which that it does I think I can demonstrate, and shall (God willing) one day make it appear to the publick.

The promised publication never appeared, and it seems that Halley never succeeded in determining the amount of this acceleration. In the second edition of the *Principia* [1713, 421], Newton did mention that Halley was the first to discover the Moon's acceleration as shown by Babylonian eclipses and eclipses observed by al-Battānī. This reference, however, was suppressed in the third edition of 1727 for reasons I have been unable to discover.[4] Moreover, Halley makes no reference to this acceleration in his lunar tables [1749], which were completed (although not published) by 1720, suggesting that he was unable to satisfy himself that it really existed.

After Halley, the question of the Moon's acceleration was taken up by Richard Dunthorne [1749, 162] who attempted to determine the amount of 'that acceleration of the Moon's motion which Dr. Halley suspected'. In his determination, he rejected eclipses observed by Tycho Brahe and Bernard Walther as being too near his own epoch, and also those observed by al-Battānī because of the uncertainty of the longitudes of Antioch and Racca. Instead, he used three solar eclipses—two of which were reported by Ibn Yūnus (977 and 978) and the other by Theon (364)—and three lunar eclipses reported by Ptolemy (−720, −382, −200). He chose the latter because each occurred near Sunrise or Sunset and thus afforded a partial check on the times reported by Ptolemy. From these eclipses, Dunthorne concluded that the magnitude of the Moon's acceleration was roughly 10″, an estimate which has proven to be very nearly correct.

Values of the accelerations similar to Dunthorne's were subsequently obtained by Mayer [1752] and Lalande [1757], but neither introduced any additional observational evidence or significantly improved upon Dunthorne's rough analysis.[5] Concurrently, the Moon's acceleration was proving an embarrassment to theoretical astronomers, since no gravitational explanation

[4] I can find no reference to this question in the published correspondence of either Halley [MacPike 1932] or Newton [Turnbull 1959–1961, Edleston 1850, Cohen 1958, Rigaud 1841].

[5] Mayer [1752, 389–392] discusses only the two Arabian eclipses used by Dunthorne and remarks on the unsatisfactory nature of the Ptolemaic eclipse reports.

for this phenomenon could be found. As a result several papers appeared, most notably by Lagrange [1773], Jean Bernoulli [1773], and LaPlace [1773], in which the authors emphasized that the empirical evidence supporting the existence of this phenomenon was not decisive, particularly in view of the (ostensibly) dubious reliability of Ptolemy's reports. Curiously, all these authors considered only the eclipses discussed by Dunthorne and ignored the 16 others described in the *Almagest*.

A theoretical explanation of the Moon's acceleration was finally achieved by LaPlace [1786], who showed that it resulted from a slow variation of the eccentricity of the Earth's orbit. Moreover, LaPlace's initial computation of the magnitude of the acceleration, 11.135″, agreed well with the empirical determinations of Dunthorne, Lalande, and Mayer.

The close agreement between the theoretical and empirical values of the Moon's acceleration reduced the suspicion with which Ptolemy's eclipse-reports had been regarded. It also reduced the necessity of a more precise empirical determination, since the magnitude of the acceleration could be computed from gravitational theory using elements known with high accuracy from modern observations. In his *Mécanique celeste*, LaPlace [Bowditch 1829–1839, iii 643] justified his final value for the Moon's acceleration, 10.18″ ..., with the remark,

> This secular equation is placed beyond doubt by Mr. Bouvard, by a profound discussion of the ancient eclipses which were known to astronomers and also of those he has obtained from an Arabian MMS of Ibn Yunis.[6]

Bouvard, however, seems not to have published this paper, and LaPlace evidently did not think it necessary to discuss his results further. Elsewhere LaPlace [1835, 492–494] showed that his own computed values of the accelerations of the Moon's elongation, anomaly, and argument of latitude yielded values for these arguments at Thoth 1, Nabonassar 1 (Ptolemy's epoch) that were in good agreement with Ptolemy's tabular values, values which LaPlace took as representative of Ptolemy's eclipse-data.

In his tables, Mayer includes a correction for the Moon's acceleration equivalent to $+6.7'' T^2$ (epoch: 1700), without indicating how he arrived at this number. In a subsequent revision of his tables, Mayer [1770] changed the magnitude of the acceleration to +9.00″, again without explanation.

Lalande [1751, 430] obtained the value of +9.886″, using the same eclipses as Dunthorne, but after making small corrections to the Moon's mean anomaly at the time of the Arabian eclipses (+977,8).

[6] A text and translation of the observations reported by Ibn Yūnus were published by Caussin in 1804.

As a result of LaPlace's work, it was generally accepted that the available ancient observations supported the magnitude of the Moon's acceleration computed from gravitational theory, which in turn was considered more accurate than any empirical determination. Consequently, ancient eclipses received little attention during the first half of the nineteenth century, except for occasional attempts [cf., e.g., Wurm 1817, Zech 1851] to improve the modern values of the Moon's mean motions in anomaly and argument of latitude.

By 1850, improvements in the accuracy of the lunar theory made it possible to use the path of totality of solar eclipses as evidence of the magnitude of the Moon's acceleration. Airy [1853, 1857], and Hansen [1854, 8] investigated the circumstances of a few ancient solar eclipses which appeared to have been total at known places, and showed that these reports could be satisfied by a small increase in the value of the secular acceleration found by LaPlace. As a result, Hansen adopted the value 12.18″ for the sidereal acceleration of the Moon in his lunar tables published in 1857, even though this value differed from the theoretical value.

Shortly before the publication of Hansen's lunar tables, Adams [1854] showed that certain terms in the development of the theoretical value of the acceleration, which LaPlace and others had neglected as insensible, were not insensible at all; and that, when these were included, the value for the acceleration was roughly half that obtained by omitting them. This discovery precipitated a heated controversy, but was eventually accepted. The definitive value for the Moon's theoretical sidereal acceleration was found by Brown [1909, 148; 1919] to equal $+6.05'' \pm 0.02''$ (1900).

By destroying the apparent agreement between the theoretical value of the secular acceleration and that found from ancient eclipses, Adams' discovery re-established the desirability of securely determining the secular acceleration from ancient observations. The problem should have been straightforward, since, as Newcomb [1878, 25] pointed out, the secular acceleration could be determined from the Ptolemaic and Arabian eclipses with a probable error of $\pm 0.4''$ and $\pm 0.8''$ respectively, if the Moon's mean centennial motion could be determined from modern observations with an equivalent accuracy. The latter seemed possible given the number and precision of observations since 1750, provided that the deviations from theory since 1750 could be attributed to either observational errors or errors in theoretical terms of short period. Thus, the principal requirements for a straightforward solution were merely that the coefficients of the significant theoretical inequalities of long period be accurate and that the ancient observations be free of large systematic errors.

As it turned out, neither requirement could be satisfied with certainty. The first condition—that Hansen's lunar theory should adequately represent the inequalities of long period in the Moon's motion—was initially challenged by Delaunay [1863], who showed that a large term which Hansen had found to arise from the action of Venus, $+21.47'' \sin(8V - 13G + 4;44°)$, was virtually insensible ($0.272''$) when its development was completed. Due to the difficulties attending the development of the planetary terms in the lunar theory, this conclusion (like Adams') was also questioned for some time. But subsequent investigations confirmed Delaunay's calculation, and virtually eliminated the possibility that a term of this magnitude would remain undetected.

Since Hansen [1854] had shown that his theory, including the questionable Venus-term, satisfied the observations from 1750 to 1850 well, the correction of this term meant that the Moon exhibited unexplained deviations from its theoretical position. These deviations, moreover, could not be adequately described by the observations in this interval, since the period of the inequality supposed to account for them (239 years) was more than twice the interval for which reliable observations were available. Thus, the determination of the secular acceleration from ancient observations came to require also a resolution of the discordance between modern theory and observations, in order to permit establishing the Moon's mean motion securely from modern observations.

Modern determinations of the accelerations of the Sun and Moon

The problem of re-determining the Moon's acceleration from ancient observations was first attacked intensively by Newcomb. In 1870, he showed that Hansen's theory, even with the erroneous Venus-term, failed to satisfy both a number of eclipses prior to 1750 and the most recent observations since 1850. This removed any possibility of describing the Moon's deviation from theory solely by means of observations from the period 1750–1850, and caused Newcomb to investigate observations of occultations and eclipses made by 17th and 18th century astronomers (later extended in his second memoir to include observations of occultations to 1908).

Having extended the interval for which lunar observations could be used to obtain the necessary corrections to Hansen's theory, Newcomb made two separate attempts to determine these corrections. The first, published in 1878, used observations of occultations and eclipses from 1620 to 1750 together with the errors deduced from Hansen's theory by eliminating the above-mentioned Venus-term. The second, published in 1912, extended the comparisons of occultations to 1908 and introduced certain corrections to

Hansen's elements and planetary terms. In both investigations, Newcomb rejected all ancient reports of ostensibly total solar eclipses,[7] and determined the Moon's acceleration from the times of the lunar eclipses reported by Ptolemy and of the lunar and solar eclipses described by Ibn Yūnus.

The results of these two investigations were very nearly identical, despite the several refinements and great amount of additional observational material included in the later paper. After removing the empirical Venus-term, Newcomb [1878] found the following corrections to Hansen's mean longitude for 1800:

$$\begin{aligned} \Delta L_{1878} &= -1.14'' - 29.17''T - 3.86''T^2 + 15.5'' \sin(1.32^\circ T + 93.9^\circ) \\ S'_D &= 2.27'' \\ S_D &= 8.30'', \end{aligned} \tag{1}$$

while in his later paper he found the correction to be:

$$\begin{aligned} \Delta L_{1912} &= -0.31'' - 26.57''T - 4.22''T^2 - 0.0067''T^3 \\ &\quad + 12.95'' \sin(1.31^\circ T + 100.6^\circ) \\ S'_D &= 1.91'' \\ S_D &= 7.94''. \end{aligned} \tag{2}$$

Subsequently, Brown [1913, 699; 1915, 513] found that Newcomb omitted some planetary terms of long period in his second paper which, when included, made Newcomb's final result for 1800:

$$\begin{aligned} \Delta' L_{1912} &= -1.14'' - 27.24''T - 3.378''T^2 - 0.0067''T^3 \\ &\quad + 12.95'' \sin(1.31^\circ T + 100.6^\circ) \\ S'_D &= 2.75'' \\ S_D &= 8.77''. \end{aligned} \tag{3}$$

In his papers of 1878 and 1912, Newcomb followed slightly different procedures in arriving at his corrections to Hansen's elements, but both solutions were based on the assumption that the deviation from theory in modern times was properly described by a mean motion and sinusoidal term which minimized the squares of the deviations. The major part of Newcomb's correction to the Moon's mean motion and his entire correction to the mean

[7] Cf. Newcomb 1878, 28–34; 1912, 228–246, for an excellent critical discussion of the quality of the ancient reports of total eclipses as evidence for determining the amount of the accelerations of the Sun and Moon.

longitude at epoch thus arise from solving the equations of condition derived from modern observations on the assumption of a periodic deviation.

Furthermore, a substantial part of Newcomb's correction to Hansen's acceleration was due to his resulting correction to Hansen's mean motion. Thus, as shown in his earlier paper, Newcomb's correction to Hansen's mean motion by itself required a corresponding correction to Hansen's acceleration of

$$\Delta S_m = -1.25''(1800)$$
$$S_m = 10.9''(1800)$$

in order to satisfy the solar eclipses of Thales (−584), Larissa (−556), and Agathocles (−309), which Hansen used. Thus, the effective difference between the secular acceleration Newcomb derived from the Ptolemaic and Arabian lunar eclipses (1878) and the acceleration satisfying these three solar eclipses was ≈ 2.1″, equivalent to roughly 20 minutes in the time of an eclipse at Ptolemy's epoch and to 35 minutes at −400.

Newcomb's work raised two important problems. The first was whether it was proper to assume that an unexplained deviation from gravitational theory in the Moon's motion was periodic over the interval for which modern observations were available and, thus, whether Newcomb's reduction of Hansen's mean motion was justified. Although there appears to be no formal justification for doing so [cf. van der Waerden 1961], the absence of a more satisfactory procedure has made it common practice to determine the Moon's mean motion by a periodic least-squares analysis, which minimizes the deviations shown by modern observations. Thus, most of Newcomb's reduction of Hansen's mean motion has been accepted.

The second problem, which Newcomb discussed in his paper of 1878, was whether the Ptolemaic and Arabian eclipses did not require a smaller value in the Moon's acceleration than that which appeared to satisfy certain ancient solar eclipses. This question became a matter of controversy even before Newcomb published his second paper and eventually occasioned a re-examination in bits and pieces of all of the relevant ancient observations.

In a series of memoirs, Ginzel [1882–1884] discussed reports of over 50 solar eclipses ranging in date from −752 to 1415. From 29 of these, he obtained corrections to Hansen's elements which slightly reduced Hansen's acceleration, but which increased his mean motion in 1800 by 9″. Ginzel also arrived at a correction to the motion of the Moon's perigee which was considered too large to fall within the limits of uncertainty of either modern theory or modern observations. Finally, in his *Spezieller Kanon der Finsternisse* [1885, 5], Ginzel published small additional corrections. In 1887, Oppolzer published his *Kanon der Finsternisse* [cf. Oppolzer 1962], which was based upon Hansen's elements modified by a different empirical correction than Ginzel's. Newcomb [1912, 238] showed that Oppolzer's correction to the Moon's mean motion and secular acceleration was virtually identical with his own, but that Oppolzer also incorporated inadmissible corrections to the mean motion of the node and the secular acceleration of the perigee, both of which were thought to be determined securely from gravitational theory.

In 1905 and 1906, Cowell analyzed reports of six ancient solar eclipses, which seemed to indicate that totality was visible at specific locations. Except for the eclipses of −309 (Agathocles) and −430 (Thucydides), neither Newcomb nor Airy had previously discussed any of these eclipses. Cowell concluded that five solar eclipses (−1062, −762, −647, −430, and 197) could be satisfied only by decreasing the secular acceleration of the Moon's node or increasing the secular acceleration of the Sun and Moon by 3.5″.

Newcomb challenged Cowell's results, arguing that such a reduction in the acceleration of the node was inadmissible on theoretical grounds, while his own analyses of modern observations of the Sun and Mercury rendered implausible the existence of a solar acceleration only a third as large as Cowell proposed. Nevertheless, although the numerical results of Cowell's analysis were never widely accepted, his suggestion that the Sun exhibited a perceptible acceleration was eventually confirmed by subsequent investigators.

After Newcomb's last memoir, Fotheringham took up the problem of determining the secular accelerations of the Sun and Moon from ancient observations. In a series of papers extending from 1909 to 1927, Fotheringham analyzed not only the observations of solar and lunar eclipses [1920a–b] which had been previously utilized for these purposes, but also the equinox-observations of Hipparchus [Fotheringham 1918], the lunar eclipse-magnitudes reported by Ptolemy [Fotheringham 1909a], and the lunar occultations reported in the *Almagest* [Fotheringham 1915a]. His final estimate of the values best satisfying the eclipses and occultations was $S_m = +10.8''$, $S_s = +1.5''$, and $S_D = 9.3''$ ($S'_D = 3.27''$), applied to a mean motion and epoch (1800) very nearly identical to Newcomb's [cf. Fotheringham 1920b, 125].

Fotheringham's values for the secular accelerations derived from different types of observations are shown in Table A1.1. His discussion of the non-Babylonian eclipses reported by Ptolemy led to nearly the same acceleration of the Moon's mean elongation as the one Newcomb had obtained from his analysis of both Ptolemaic and Arabian eclipses. His investigations of other ancient data, however, indicated both a larger secular acceleration of the Moon and the existence of a sensible acceleration of the Sun. The latter was perhaps Fotheringham's most significant finding, and was attested directly by the Alexandrian eclipse-magnitudes and Hipparchus' equinox-

	Lunar Eclipses	Lunar Eclipse Magnitudes	Occultations	Equinoxes (Hipparchus)	Solar Eclipses (Totality)
S_m			$10.3'' \pm 0.74''$ [a]		$10.8''$
S_s		$1.78'' \pm 0.45''$		$1.95 \pm 0.27''$	$1.5''$
S_D	$7.9''$				$9.3''$

[a] Corrected from 10.8″ in accordance with Fotheringham 1923, 273.

Table A1.1. Fotheringham's Accelerations of the Sun and Moon from Different Ancient Observations

observations, as well as indirectly by the difference between the values for the lunar acceleration derived from occultations and the acceleration in elongation derived from lunar eclipses.

The individual values for the Sun's acceleration determined from the different sets of observations were not entirely consistent, and the discrepancies appeared to support a relatively high value for this acceleration. The occultations and lunar eclipses suggested a solar acceleration of 2.4″ (originally 2.9″, close to Cowell's value), compared with roughly 1.9″ (originally 1.0″) from equinoxes, 1.8″ from eclipse-magnitudes and 1.5″ from solar eclipses. Similarly, the acceleration of the Moon's elongation found from occultations, equinoxes, and eclipse-magnitudes was 8.4″, compared with 9.3″ from solar eclipses and 7.9″ from lunar eclipses. Thus, Fotheringham's results appeared to confirm the discrepancy, first suggested by Airy [1853] half a century earlier, between the acceleration in elongation implicit in the lunar eclipse-times and that derived from other ancient data.

Fotheringham's results became an important element in the derivation of the accelerations presently accepted as 'modern theory'. Accordingly, the specific values which he obtained from different types of observations deserve critical scrutiny.

First, in determining the Sun's secular acceleration from Hipparchus' equinoxes, Fotheringham [1918] assumes a constant error in declination (−0;4.4°), which he derives from Hipparchus' declinations of seven stars near the equator [cf. Ptolemy, *Alm.* vii 3]. He then applies this error, which differs appreciably from the mean systematic error of +0;0.7° for all 18 declinations [cf. Pannekoek 1955, 64], to Hipparchus' spring equinoxes from −134 to −127 in order to obtain his 'definitive result',

$$S_s = +1.95'' \pm 0.27''.$$

In the same paper, Fotheringham showed that assuming an error in declination which would yield the best fit for all equinoxes (−0;7.6° ± 0;0.46°) would make the most probable acceleration

$$S_s = +1.0'' \pm 0.18''.$$

Thus, while the probable errors obtained from the discordances are relatively small, the determination is very sensitive to the assumed systematic error in declination. On balance, the lower result seems at least as probable as the higher, but virtually any value for the secular acceleration of the Sun between $\approx +0.8''$ and $+2.0''$ is arguably consistent with Hipparchus' equinox-observations.

Much the same can be said of Fotheringham's determinations based on the reported lunar eclipse-magnitudes and occultations. In the case of the former, he [1909a] excludes the Babylonian eclipses, which would increase the secular acceleration, while taking no account of the uncertainty of the motion of the node. As a result his final determination,

$$S_s = +1.78'' \pm 0.45'',$$

is uncertain by a considerably larger amount than the error he estimates.

In the case of the occultations, the result, which Fotheringham deduced from a set of seven very discordant observations, depends largely on his assumptions about the probable clock-errors. Using three different assumptions—(1) that the clock-error was proportional to the time from Sunrise or Sunset, whichever was closer to the event; (2) that the clock-error was independent of the time from Sunrise or Sunset; and (3) that the clock-error was proportional to the time from Sunset alone (which he describes as 'improbable,')—Fotheringham [1915a, 393] found:

$$\text{(a)}\ S_m = +10.8'' \pm 0.7''$$
$$\text{(b)}\ S_m = +10.8'' \pm 0.9''$$
$$\text{(c)}\ S_m = +10.0'' \pm 0.8''.$$

Of these, he accepted (a) as the most probable. Subsequently, Fotheringham [1923] corrected an error in his comparisons, thereby modifying the above values (assuming the same modern mean motions) to:

$$\text{(a}'\text{)}\ S_m = +10.1'' \pm 0.7''$$
$$\text{(b}'\text{)}\ S_m = +10.1'' \pm 0.9''$$
$$\text{(c}'\text{)}\ S_m = +\ 9.3'' \pm 0.8''.$$

From (a′) and a further correction to Cowell's value for the Moon's mean motion, Fotheringham [1920b, 125] concluded that the Moon's sidereal acceleration best satisfying Ptolemy's occultations was

$$S_m = 10.3'' \pm 0.74''.$$

The probable error of this, however, could easily be increased by a slightly different estimation of the probability of assumptions (a) and (c).

In 1920, at the conclusion of a paper re-investigating the ancient solar eclipses, Fotheringham [1920b, 126] announced his oft-quoted values for the secular accelerations of the Sun and Moon,

$$S_m = +10.8'' \quad S'_m = +4.75''$$
$$S_s = +\ 1.5'' \quad S'_D = 3.25'',$$

which he asserted best satisfied all classes of ancient data. As shown by the graph on [1920b, 123] of that paper, these eclipses give extremely uncertain and discordant results. Indeed, Fotheringham seems to have obtained his final values by assuming the value of the secular acceleration of the Moon previously derived from the Ptolemaic occultations (10.8″), and accepting the largest solar acceleration consistent with this value and the condition that the eclipse of −128 be total at the Hellespont. His subsequent correction of the Moon's acceleration as determined from the occultations would have satisfied the eclipse of −128, with values for the solar acceleration ranging from +0.9″ to +1.25″; while his lower value for the Moon's acceleration derived from the occultations under assumption (c) would have satisfied the eclipse of Hipparchus, together with several others with a solar acceleration ranging from +0.5″ to +0.9″.

If we disregard Fotheringham's determination of the Sun's acceleration from eclipse-magnitudes and Hipparchus' equinoxes as too uncertain, or, alternatively, if we accept the value $S_s = +1.0''$ derived from his intitial analysis of the equinoxes as equally probable as his concluded value (+1.95″), then the bulk of the solar eclipses, including that of Hipparchus, would be satisfied by the accelerations:

$$S_m = +9.9'' \pm 0.4''$$
$$S'_m = +3.85'' \pm 0.4''$$
$$S_s = +0.9'' + 0.2''$$ [8]
$$S'_D = +2.95'' \pm 0.6''.$$

These values, moreover, agree with the corrected results of Fotheringham's analysis of the occultations on either assumption concerning the clock-errors, as well as with his initial determinations of the secular acceleration

[8] Cf. Fotheringham 1923, 123.

of the Sun from Hipparchus' equinoxes. They also agree very nearly with Newcomb's final determination (as corrected by Brown) of the acceleration of the Moon's elongation from both Arabian and Ptolemaic eclipses, the discordance being reduced to $\approx 0.2''$.

Following Fotheringham's investigations, Schoch [cf. 1926, 3; 1931] recomputed the occultations described in the *Almagest* with greater precision than Fotheringham had and also re-investigated the circumstances of a number of ancient solar eclipses. Schoch's procedure for determining the values of the two accelerations from this material contrasted sharply with both Newcomb's and Fotheringham's. Whereas they had derived their results from the average deviations of a relatively large number of observations, Schoch's values, as far as I can make out, were determined from two events, the occultation of Spica observed by Timocharis in −282 Nov 8 [Ptolemy, *Alm.* vii 3: Toomer, 336] and the solar eclipse of −128 Nov 20 associated with Hipparchus. Concerning the former, Schoch noted a discrepancy (previously remarked by Ptolemy) between the time reported for the occultation and the comment that it occurred 'just as the Moon was rising'. Accepting the second designation as more accurate and interpreting it to mean that the occultation took place half an hour after Moonrise, Schoch concluded that the sidereal secular acceleration of the Moon was

$$S_m = +11.09''.$$

Although he gives no details, he says in the same work [1926, 3] that the Sun's acceleration was determined from the ancient solar eclipses, of which 'the best criterion for [determining] the element is the eclipse of Hipparchus in −128'. Since Schoch's adopted value, $S_s = +1.511''$, would make this eclipse central at the Hellespont, given the lunar acceleration noted above, his result appears to rest on this assumption.

Having determined the accelerations in this manner, Schoch [1926, 2] dismissed the lunar eclipses reported by Ptolemy as 'worthless', and showed that his values agreed more or less with various solar eclipse-reports and with a lunar eclipse in −424 Oct 9 recorded in a cuneiform text [Kugler 1913–1935, 233]. Since both Fotheringham and Newcomb showed that some eclipses can always be more or less satisfied by any pair of reasonable accelerations, Schoch's procedure scarcely enhances the credibility of his results. In this respect, it is also unfortunate that Schoch did not publish more of the details of his computations and comparisons.

The results obtained by Fotheringham and Schoch were further analyzed by de Sitter in a paper published in 1927, which was generally accepted by contemporary astronomers as the definitive discussion of the ancient observational evidence. In it de Sitter sets up separate equations of conditions for:

(i) the accelerations of the Sun determined by Fotheringham from
  (a) Hipparchus' equinoxes,
  (b) the solar eclipses, and
  (c) the lunar eclipse-magnitudes;
(ii) the accelerations of the Moon determined from
  (d) Ptolemy's occultations and
  (e) ancient solar eclipses;
(iii) the relationship between the two accelerations found by Fotheringham from
  (f) the eclipse of Hipparchus (−128); and
(iv) the acceleration of the Moon's elongation determined by Fotheringham from
  (g) the Alexandrian lunar eclipses and
  (h) Schoch's discussion [1926, 3] of the Babylonian lunar eclipse of −424.

After weighting these equations according to Fotheringham's and Schoch's estimates of the probable error of each determination, de Sitter [1927, 23] obtained the non-gravitational accelerations (1900),

$$S'_m = (5.22'' \pm 0.30'')R$$
$$S_s = (1.80'' \pm 0.16'')R,$$

where $R = T^2 + 1.33T - 0.26$. R was introduced to minimize the effect of the corrections on the agreement between theory and modern observations, and makes the effective epoch of the mean motions 1833.5.

De Sitter's procedure in arriving at these results affords several grounds for criticism, and it is hard to understand why others have accepted his analysis so uncritically as representing the evidence of ancient observations. In the first place, he treats a number of Fotheringham's results—e.g., the accelerations of the Sun and Moon derived from solar eclipses, and the relation between them derived from the solar eclipse of Hipparchus (−128)—as independent determinations, when in fact they are independent neither of each other nor of the rest of Fotheringham's results. Indeed, the only evidence afforded by the solar eclipses alone which supports the relatively high value for the lunar acceleration adopted by Fotheringham is the so-called Eclipse of Babylon in −1062. Since there is considerable doubt as to whether this vague report refers to an eclipse at all [cf. Fotheringham 1920b, 105–106], there is no justification for counting it a condition to be satisfied.

A second criticism of de Sitter's procedure is that he adopts Fotheringham's estimates of probable error as the basis of weighting his equations without taking any account of the sensitivity of Fotheringham's results to slightly different assumptions about the observational procedures or their possible systematic errors. This is particularly true of the values of the secular acceleration of the Sun determined from Hipparchus' equinox-observations and from lunar eclipse-magnitudes and of the lunar acceleration determined from occultations.

Finally, and most significantly, de Sitter's results are vitiated by important numerical errors. In deriving the equation of condition for the Moon's secular acceleration as determined from the occultations—which is the only independent evidence in support of a lunar acceleration greater than 10″—de Sitter not only disregards Fotheringham's subsequent correction of his first determination, he also computes ΔL incorrectly, arriving at a figure 610″ too large. Even worse, in his equations derived from the accelerations of the Moon's elongation found by Fotheringham, he includes the total difference, $S_D = S_m - S_s$, into the computation, although the rest of his equations and his solution are for only the non-gravitational component, S'_D. To correct for this, the numbers +2950″ and +2320″ [de Sitter 1927, 22] must be replaced by +620″ and +660″, respectively.

When these corrections are made and de Sitter's weights for individual equations of conditions are revised to reflect somewhat larger estimates of the probable errors in each determination than Fotheringham's, significantly lower values for both accelerations result. Furthermore, de Sitter's use of Fotheringham's revised determination of the Sun's acceleration from Hipparchus' equinoxes (1.95″) instead of his initial solution (1.0″) seems unjustified in view of the several questionable assumptions which Fotheringham made in arriving at the higher value. Although these observations are at best tenuous evidence of the magnitude of the Sun's acceleration, it seems preferable to accept the lower value with a probable error equal to roughly the same amount (±1.0″) in combining determinations from different types of observations.

With these corrections, and using the mean of Fotheringham's corrected results for the occultations deduced from assumptions (a′) and (c′) [see 164, above], I find on re-solving de Sitter's equations:

$$
\begin{aligned}
S_m &= +9.67'' \pm 0.5'' \\
S'_m &= +3.62'' \pm 0.5'' \\
S_s &= +1.14'' \pm 0.3'' \\
S_D &= +8.53'' \pm 0.6'' \\
S'_D &= \ 2.48'' \pm 0.6''.
\end{aligned}
$$

These values satisfy all of the Ptolemaic observations; and the acceleration of the Moon's elongation, S_D, is very close to what Newcomb deduced from Ptolemaic lunar eclipses. To satisfy the majority of the ancient solar eclipses discussed by Fotheringham [1920b] would require that $S_D = 8.9''$ and, thus, either a somewhat larger lunar acceleration ($\approx +10.1''$) or a smaller solar acceleration ($\approx +0.8''$); but the uncertainties and ambiguities attending these reports greatly diminish their value as evidence of either acceleration [cf. Newcomb 1912, 228–246]. Furthermore, the Arabian eclipse reports discussed by Newcomb are best satisfied by opposite corrections, namely, an increase in the Sun's acceleration or a decrease in the Moon's acceleration. Since these eclipses are nearer the modern epoch, and since there are difficulties with some of the reports as well as systematic differences among observations made by different observers, they cannot be taken as conclusive evidence. Nevertheless, they seem at least as valuable as the ancient reports of total solar eclipses and so tend to offset the evidence of the latter.

De Sitter's paper [1927] also addressed the correlation between the apparent accelerations and fluctuations (unexplained discrepancies between observations and gravitational theory) in the longitudes of the Sun, Moon, and planets. If these are due entirely to variations in the Earth's rotation, then their magnitudes should be in proportion to their mean motions. He found this to be true for the accelerations and fluctuations of the Sun and inner planets, clearly not true in the case of the Moon's acceleration, and unclear with respect to the Moon's fluctuations.

After removing the effects of the accelerations derived from ancient observations, de Sitter compared the total fluctuations (including Newcomb's 'great empirical term') of the Sun, Moon, Mercury, and Venus. He found that the best solution to the residuals gave

$$Q\ {}^{n_i}/_{n_m} = 1.25\,{}^{n_i}/_{n_m}$$

as the most probable ratio of the magnitudes of the fluctuations of the Sun and planets to those of the Moon (here n_i is the mean motion of the Sun or planet in question and n_m that of the Moon).

Subsequently, in 1939, Morgan and Scott demonstrated that the meridian-observations of the Sun from 1900 to 1937 could be satisfied by assuming $Q = 1.00$. In the same year, Spencer Jones [1939] reviewed the entire body of modern observations of the Sun, Moon, Mercury, and Venus. Using de Sitter's value for the non-gravitational acceleration of the Moon, $+5.22''$, Jones first solved the equations of conditions for Q and the Sun's acceleration, obtaining

$$Q = 1.025 \quad S_s = +1.25''$$

and

$$Q = 1.062 \quad S_s = +1.26'',$$

depending on whether observations of the Sun's right ascension were included in the analysis. From these results, Spencer Jones concluded that Q was indeed unity.

Re-solving for $Q = 1.00$, Spencer Jones found for the Sun's acceleration

$$S_s = +1.14'' \pm 0.11'' \quad \text{(from solar observations)} \tag{4}$$

and

$$Ss = +1.24'' \pm 0.04'' \quad \text{(from Mercury transits),} \tag{5}$$

giving a weighted mean of

$$S_s = +1.23'' \pm 0.04''. \tag{6}$$

These values depend upon the assumption that de Sitter's value for the non-gravitational acceleration of the Moon, $+5.22''$, represents the actual non-gravitational acceleration of the Moon over the period for which modern observations are available. As Spencer Jones pointed out, any change ($\Delta S'_m$) in this value would require a corresponding change,

$$\Delta S_s = \frac{n_s}{n_m} \Delta S'_m = 0.0747 \Delta S'_m,$$

in the value of the secular acceleration of the Sun to satisfy the condition that $Q = 1$ for the fluctuations.

In discussing the discrepancy between his results and de Sitter's, Jones determined the value of $\Delta S'_m$ which would give the same ratio between the non-gravitational accelerations of the Moon and Sun as de Sitter's values. He concludes [1939, 555–556],

> The best values that we can assign for the (non-gravitational) secular accelerations of the Sun and Moon at the present time (or more strictly the best average values for the past two hundred and fifty years) are therefore:
>
> For the Moon $\quad S'_m = +3.11'' \pm 0.57''$
>
> For the Sun $\quad S_s = +1.07'' \pm 0.06''.$
>
> These values of the accelerations will not satisfy any of the ancient observations of eclipses and occultations, which are on the whole in very good agreement with each other in requiring appreciably larger values. There seems to be no escape from the conclusion that the effects of tidal friction are appreciably less at the present time than the average effects over the past two thousand years.

In view of the errors in de Sitter's analysis, and of the evidence discussed above that most of the ancient observations are well satisfied by lower values for both the lunar and solar accelerations than de Sitter found, Spencer Jones' conclusion seems untenable. Indeed, if we replace de Sitter's published value for the non-gravitational acceleration of the Moon ($+5.22''$) with that found by re-solving his equations with appropriate corrections [see 168, above], the secular accelerations from Jones' analysis become:

$$\begin{aligned} S'_m &= 3.62'' \pm 0.5'' \quad \text{(de Sitter revised)} \\ S_s &= 1.11'' \pm 0.06'' \quad \text{(S. Jones revised)} \\ S_D &= 2.51'' \pm 0.5'' \quad \text{(S. Jones revised)} \end{aligned} \tag{7}$$

Alternatively, if we assume, following Spencer Jones, that the ratio of the accelerations has remained constant (i.e., $^{3.62}/_{1.14}$), we obtain:

$$\begin{aligned} S'_m &= 3.50'' \pm 0.5'' \\ S_s &= 1.10'' \pm 0.06'' \end{aligned} \tag{8}$$

Both sets of values, (7) and (8), are in excellent agreement with those found from re-solving de Sitter's equations with correct data and revised weights. Thus, the apparent discrepancy between the accelerations determined from ancient and modern observations arose mainly from errors committed by de Sitter and unwittingly introduced into accepted theory by Spencer Jones.

In 1948, Clemence transformed Jones' non-gravitational acceleration of the Sun to an expression for ΔT, being the difference between observed Universal Time and an invariable Ephemeris Time (originally called Newtonian time by Clemence). Expressed in Ephemeris Time (ET) Jones' non-gravitational acceleration of the Moon becomes

$$\frac{1}{2}\dot{\eta}_m = 5.22 - 13.368 \cdot 1.23'' = -11.22''$$

or

$$\dot{\eta}_m = -22.44\,''/\text{cy}.$$

Interestingly, the first person to publish an analogous calculation of the Moon's secular retardation was Schoch [1926, 34] who found

$$\frac{1}{2}\dot{\eta}_m = -14.84'' \qquad \text{or} \qquad \dot{\eta}_m = -29.68\,''/\text{cy}.$$

In 1952, Spencer Jones' and Clemence's accelerations were adopted by the International Astronomical Union, and they have since been incorporated in the ephemerides prepared by the American and British Nautical Almanac Offices [1961, 94]. Thus did Fotheringham's results and de Sitter's errors become part of modern theory.

Recent investigations of the secular accelerations

Since the adoption of Spencer Jones' accelerations in 1952, there have been several further studies of these parameters, which on the whole have left the matter as uncertain as ever. In 1952, Brouwer revised and extended Spencer Jones' analysis, excluding de Sitter's results and using modern data from lunar occultations through 1948 and the results of Newcomb's analysis of Ptolemaic and Arabian eclipse-times for ancient data. From these, he obtained

$$\begin{aligned} S_s &= 1.01'' \quad \text{(epoch: 1715)} \\ S'_m &= 2.22'' \quad S'_D = 1.21'' \end{aligned} \tag{9}$$

as the accelerations best fitting the ancient and modern data, although he noted [Brouwer 1952, 141] that this result is sensitive to how the Moon's (modern) fluctuations are treated. Brouwer showed that these appeared to be random instead of periodic, and his solution was based on this premise.

In 1961, van der Waerden extended Brouwer's methodological discussion and tried to reconcile the observed accelerations with Jeffreys' theory [1952, 225] which suggested that the ratio of the apparent accelerations, S_m/S_s, should be roughly 6.9, far higher than that resulting from Spencer Jones' accelerations (4.2), let alone Brouwer's (2.2). Van der Waerden derived revised accelerations from four data-points having mean epochs of: 1962 (based on extrapolations from 1958.0 lunar data; 1635 (based on Newcomb's analysis of observations by Gassendi and Hevelius); 950 (based on Brouwer's data derived from Newcomb's study of Arabian eclipse-times); and −386 (based on his own analysis of three apparently critical ancient observations). These last were (a) the Babylonian lunar eclipse of −424 Oct 9 [cf. also Schoch 1926; de Sitter 1927]; (b) the lunar eclipse of −382 Dec 23 observed in Babylon and reported by Ptolemy [see 61–63, above]; and (c) the lunar occultation observed by Timocharis on −282 Nov 8 [see 86–88, above]. These three observations give very discordant results, and van der Waerden's result does not represent any one of them very well, let alone all three. Nevertheless, from these data, he finds accelerations of:

$$\begin{aligned} S_s &= 1.31'' \pm 0.10'' \quad \text{(epoch: 1755)} \\ S'_m &= 6.28'' \pm 0.82'' \\ S'_D &= 4.97'' \pm 0.9''. \end{aligned} \tag{10}$$

In 1966, Curott investigated ancient records of solar eclipses using Ephemeris Time and Spencer Jones' (de Sitter's) value for the Moon's acceleration (5.22″) together with other modern parameters. He found an apparent solar acceleration of 1.10″ ± 0.06″ (epoch: 1900), which becomes

$$S_s = 1.25'' \pm 0.07'' \quad \text{(epoch: 1780)},$$

a result virtually identical with Jones'. Curott, moreover, found an average value for $^{\Delta S_s}/_{\Delta S_m}$ of 0.12 for the relevant eclipses, so that for

$$\Delta S'_m = -1.6'' \ \pm 0.5'',$$
$$\Delta S_s = -0.19'' \pm .06''$$

or

$$\begin{aligned} S'_m &= 3.62'' \pm 0.5'' \\ S_s &= 1.06'' \pm 0.09'' \quad \text{(epoch: 1780)}, \end{aligned} \tag{11}$$

a result virtually identical with Spencer Jones' as revised [cf. 171, above].

In 1969, R. R. Newton announced that he had re-analyzed all of the traditional (i.e., non-cuneiform) ancient and medieval observations and found that the apparent accelerations of both the Sun and Moon varied significantly with time. In particular, he found the following (average) accelerations since 1900 for ancient and medieval observations, respectively:

	Before 500 Epoch: −200	After 500 Epoch: 1000
$S_s =$	$1.79'' \pm 0.22''$	$1.45'' \pm 0.23''$
$S'_m =$	$1.34'' \pm 2.15''$	$3.12'' \pm 3.05''$
$\dot{\eta}_m =$	$-41.6'' \ \pm 4.3''$	$-42.3'' \ \pm 6.1''.$

(12)

Comparing these with a value for $\dot{\eta}_m = -20.1'' \pm 2.6''$, which he [1969, 826] had previously found by analyzing modern data, Newton [1970, 280] concluded that (the average effective value of) $\dot{\eta}_m$ varied in time as

$$\dot{\eta}_m = -22'' + 3.3''\,T + 0.114''\,T^2 \quad (T_0 = 1900),$$

and, thus, that there was a 'strong presumption that $\dot{\eta}_m$ has changed by a factor of 2 within historical times'.

Newton's full analysis was published in 1972. It was followed in the same year by an analysis of 379 additional medieval solar eclipses, from which he found accelerations of:

$$\begin{aligned} S_s &= \quad 2.75'' \pm 0.65'' \\ S'_D &= \quad 5.32'' \pm 7.9'' \\ \dot{\eta}_m &= -78.9'' \ \pm 15.9''. \end{aligned} \tag{13}$$

These have an effective epoch of 976, and are clearly inconsistent with the values shown above, values which Newton found from Islamic observations around the same date. Subsequently, Newton implicitly abandoned both sets of results.

In 1975, Muller and Stephenson carefully investigated the circumstances of 25 reports of solar eclipses from ancient and medieval times. From these they found accelerations equivalent to:

$$\begin{aligned} S_s &= 1.88'' \pm 0.21'' \quad \text{(epoch: 1770)} \\ S'_m &= 6.32'' \pm 02.5'' \\ S'_D &= 4.44'' \pm 02.5''. \end{aligned} \tag{14}$$

Of the 25 eclipses, however, the authors regarded only seven as certain while only two contributed evidence defining the lower boundary of S_s. Of these two, one was a partial eclipse observed at an inferred location in China in 120, and the other was a total eclipse observed near the Kerulen River by the party of Ch'ang-ch'un in 1221 [Muller and Stephenson 1975, 491–493].

Furthermore, in 1975, Morrison and Ward re-investigated all of the transits of Mercury from 1677 to 1973. Assuming Spencer Jones' value for the apparent solar acceleration, they [1975, 197–198] found:

$$\begin{aligned} S_s &= \quad 1.23'' \quad \text{(S. Jones)} \\ S'_m &= \quad 3.45'' \pm 2'' \\ S'_D &= \quad 2.22'' \pm 2'' \\ \dot{\eta}_m &= -26.0''. \end{aligned} \tag{15}$$

This result is close to Spencer Jones' when the latter is adjusted to correct for de Sitter's errors, and supports the assumption of constant accelerations since ancient historical times.

Following his polemic against Ptolemy, Newton [1979–1984] attacked Jones' methodology and concluded that the solar and lunar accelerations at the modern epoch (1900) were radically different from those derived by Jones. In addition, he concluded that $\dot{\eta}_m$ is probably constant and equal to $-28.4'' \pm 5.7''$ (in contrast to his earlier finding), but that the rate of the Earth's rotation exhibits a sensible acceleration which he attributed mainly to a change in the gravitational constant. In a subsequent work, Newton [1985b, 324] found S_s to vary as

$$S_s = 0.70'' - 0.0668''T - 0.0015''T^2 \quad (T_0 = 1900). \tag{16}$$

This combined with the value $\dot{\eta}_m = -28''$ results in the following parameters for ancient and modern epochs:

$$\begin{array}{rcc} & \text{Epoch: } -300 & \text{Epoch: } 1900 \\ S_s = & 1.44'' & 0.62'' \\ S'_m = & 5.25'' & -5.71'' \\ S'_D = & 3.81'' & -6.1'' \end{array} \tag{17}$$

While these are inconsistent with his earlier findings, it is interesting that Newton's most recent accelerations for −300 are very similar to those found by Fotheringham and Schoch.

Recently, a number of investigators have used different techniques to measure the lunar acceleration ($\dot{\eta}_m$) directly. As summarized by Stephenson and Morrision [1984, 50], the most accurate of these are:

Investigator	Method	$\dot{\eta}_m$ ″/cy
Morrison/Ward [1975]	transits of Mercury	-26.0 ± 2.0
Lambeck [1980]	numerical tidal model	-29.6 ± 3.1
Cazenave [1982]	artificial satellites	-26.1 ± 2.9
Dickey/Williams [1982]	lunar laser ranging	-25.1 ± 1.2

These results suggest that the current value of $\dot{\eta}_m$ lies between $-24''$ and $-26''$, which compares favorably with the value of $-23.2''$ derived from de Sitter's analysis of ancient observations as corrected.

Stephenson and Morrison [1984] and Newton [1985b] have published new attempts to describe the variation of the Earth's rotation, assuming a constant value for $\dot{\eta}_m$ and using ancient and medieval observations incorporating extensive Babylonian data from cuneiform sources. Though their methods and conclusions differ, they all find that a constant acceleration will not account well for both ancient and medieval observations. For −300, the accelerations implicit in their studies, assuming $\dot{\eta}_m = -25''$, are:

$$\begin{array}{rcc} & \text{Stephenson/Morrison} & \text{Newton} \\ S_s = & 1.26'' & 1.32'' \\ S'_m = & 4.33'' & 5.13'' \\ S'_D = & 3.07'' & 3.81'' \end{array} \tag{18}$$

Investigator	S_s	S'_m	S'_D	$\dot{\eta}_m$	Epochs
van der Waerden [1961]	1.31″	6.28″	4.97″	−22.4″	−380/1780
Muller/Stephenson [1975]	1.88	6.32	4.44	−37.6	0/1770
S. Jones [1939]	1.23	[5.22]	3.99	−22.4	−200/1780
Newton [1985][a]	1.32	5.13	3.81	[-25.0]	−300/1790
de Sitter [1927]	1.80	5.22	3.42	−37.7	−200/1833
Schoch [1926]	1.51	5.04	3.53	−30.3	−200/1800
Fotheringham [1920b]	1.50	4.75	3.25	−30.6	−250/1800
Stephenson/Morrison [1984][a]	1.26	4.33	3.07	[-25.0]	−300/1900
Newcomb [1912][a]	[1.23]	[3.97]	2.75	[-25.0]	−300/1800
Curott [1966]	1.06	[3.62]	2.56	−21.1	≈ 0/1780
S. Jones (revised)[b]	1.11	[3.62	2.51	−22.4	−200/1780
de Sitter (revised)[c]	1.14	3.62	2.48	−23.2	−200/1833
Morrison/Ward [1975]	[1.23]	3.45	2.22	−26.0	1677/1973
Newton [1970]	1.79	3.13	1.34	−41.6	−200/1900
Brouwer [1952]	1.01	2.22	1.21	−20.5	−300/1900

[a] Calculated from $\dot{\eta}_m$ and S'_D. [b] Cf. 171, above. [c] Cf. 169, above.

Table A1.2. Summary of Recent Determinations of the Accelerations of the Sun and Moon

The results of the investigations discussed above, beginning with Newcomb [1912], are summarized in Table A1.2. Since, for ancient observations, the acceleration in elongation (S'_D) is the best determined parameter, the findings are listed in order of S_D'. Parameters which are assumed from other studies and not independently derived are shown in [ ]. More than half ($^7/_{13}$) the results give values for S_D between ≈ 2.5″ and 3.5″, with the values of S_s falling between roughly 1.1″ and 1.5″. At present, the best estimates of the (average) accelerations for −300/1900 seem to be:

$$\begin{aligned} S_s &= \ 1.15'' \pm 0.15'' \\ S'_D &= \ 2.85'' \pm 0.5'' \\ S'_m &= \ 4.00'' \pm 0.6'' \\ \dot{\eta}_m &= 25'' \pm 2''. \end{aligned} \tag{19}$$

These are very close to Stephenson and Morrison's implicit findings [1984] and to Newcomb's results [1912] when adjusted for the Sun's acceleration.

Elements of the Sun and Moon used in this work

When this study was first completed, the accelerations which seemed to fit the ancient and medieval data best were:

$$\begin{aligned} S_s &= 1.0'' \\ S'_m &= 3.62'' \quad (S_m = 9.76'') \\ S'_D &= 2.62'' \quad (S_D = 8.67''), \end{aligned} \qquad (20)$$

and these parameters were adopted in this work. Recently, the combination of better modern techniques for estimating $\dot{\eta}_m$ and the use of more extensive Babylonian data in estimating S_s have suggested that slightly higher accelerations may in fact apply. These would affect the calculated times of lunar phenomena reported by Ptolemy by no more than 10 minutes. In view of the uncertainties which still attend the values of these parameters, I have left unchanged the parameters originally adopted.

The following table shows the corrections to the *computed* times of the solar (Δt_s) and lunar (Δt_D and Δt_m), phenomena which would result from the use of the accelerations shown in (19) in place of the elements adopted in this work.

Epoch	Δt_s	Δt_D	Δt_m
140	-19^m	-3^m	-4^m
−140	−25	−3	−5
−250	−28	−4	−5
−500	−35	−5	−17
−750	−43	−5	−18

The adopted accelerations differ from those deduced by S. Jones [1939] and included in the elements accepted by the Nautical Almanac Offices by:

$$\begin{aligned} \Delta S_m &= -1.6'' \\ \Delta S_s &= -0.23''. \end{aligned}$$

These corrections should be multiplied by $R = T^2 + 1.33T - 0.26$ to minimize their effect on modern observations. Hence, the total corrections to the expressions for the mean longitudes of the Sun and Moon at 1900 become:

$$\begin{aligned} \Delta L_m &= +0.42'' - 2.13''T - 1.6''T^2 \\ \Delta L_s &= +0.06'' - 0.31''T - 0.23''T^2. \end{aligned}$$

Applying these to the elements used by the Nautical Almanac Offices [1961, 98, 107], expressed in terms of Universal Time,[9] we obtain for 1900.0:

$$L_m = 270;26,16.78^\circ + 1336^r 306;53,36.89^\circ T + 10.76'' T^2$$
$$L_s = 279;41,49.10^\circ + 100^r 0;46,10.80^\circ T + 2.09'' T^2$$
$$D = 350;44,27.68^\circ + 1236^r 307;7,26.09^\circ T + 8.67'' T^2.$$

In this work, the longitudes of the Moon's perigee and node are from the expressions derived by Brown [1915] and used by the Nautical Almanac Offices [1961, 107]. For reference, these are (1900.0):

$$P_m = 334;19,46.40^\circ + 11^r 109;2,2.52^\circ T - 37.12'' T^2$$
$$N_m = 259;10,59.79^\circ - 5^r 134;8,31.23^\circ T + 7.48'' T^2.$$

---

[9] The elements stated in Nautical Almanac Offices [1961, 98, 107] are for Ephemeris Time. To obtain expressions for the elements for Universal Time the following corrections must be applied:

$$\Delta L_m = +4.65'' + 12.96'' T + 5.22'' T^2 + B$$
$$\Delta' L_s = +1.00'' + 2.97'' T + 1.23'' T^2 + 0.0747 B,$$

where B is the value of the Moon's fluctuation. In the present study, B has been neglected because its magnitude at ancient epochs is unknown.